

Example 8: Let  $A = \begin{bmatrix} -4 & 3 & 2 \\ 2 & -6 & 2 \\ 2 & 3 & -4 \end{bmatrix}$ . If  $A$  is invertible, calculate  $A^{-1}$ . If not, explain why.

$$[A | I_3] = \left[ \begin{array}{ccc|ccc} -4 & 3 & 2 & 1 & 0 & 0 \\ 2 & -6 & 2 & 0 & 1 & 0 \\ 2 & 3 & -4 & 0 & 0 & 1 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[ \begin{array}{ccc|ccc} 2 & -6 & 2 & 0 & 1 & 0 \\ -4 & 3 & 2 & 1 & 0 & 0 \\ 2 & 3 & -4 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{R_1 := \frac{1}{2}R_1} \left[ \begin{array}{ccc|ccc} 1 & -3 & 1 & 0 & \frac{1}{2} & 0 \\ -4 & 3 & 2 & 1 & 0 & 0 \\ 2 & 3 & -4 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\begin{array}{l} R_3 := R_3 - 2R_1 \\ R_2 := R_2 + 4R_1 \end{array}} \left[ \begin{array}{ccc|ccc} 1 & -3 & 1 & 0 & \frac{1}{2} & 0 \\ 0 & -9 & -6 & 1 & 2 & 0 \\ 0 & 9 & -6 & 0 & -1 & 1 \end{array} \right]$$

$$\xrightarrow{R_3 := R_3 + R_2} \left[ \begin{array}{ccc|ccc} 1 & -3 & 1 & 0 & \frac{1}{2} & 0 \\ 0 & -9 & -6 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array} \right]$$

Not possible

Rank(A) = 2  $\neq$  n

Thus  $A$  is not invertible  
by FTIM.